Parabolas – Problems

1. $P = xy$ and $2y = 80 - 4x$.

   (a) Show that $P = 40x - 2x^2$

   (b) Find the value of x for which P is maximised.

   (c) Find the maximum value of P.

2. $A = mn$ and $6m + 3n - 90 = 0$

   (a) Show that $A = 30m - 2m^2$

   (b) Find the maximum value of A and the corresponding value of m.

3. $T = uv + u^2$ and $6u + 2v = 24$

   (a) Show that $T = 12u - 2u^2$

   (b) Find the maximum value of T.

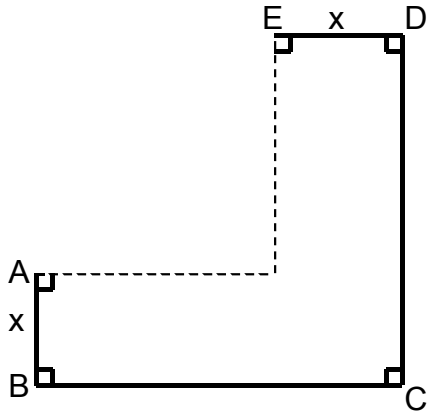
4. The shape opposite has a perimeter of 12 metres.

   AB = ED = x metres and BC = DC.

   (a) Show that $BC = 6 - x$

   (b) Hence show that the area of the shape is $A = 12x - 3x^2$

   (c) Calculate algebraically the maximum area of the shape.



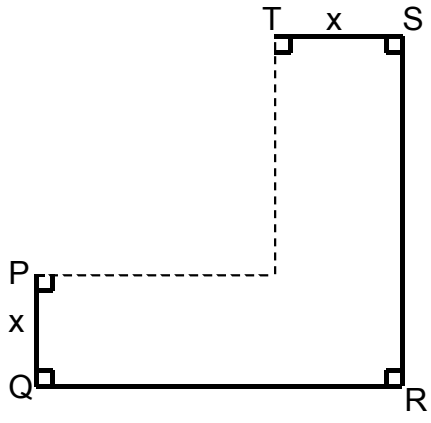
5. The diagram opposite shows a flower-bed with perimeter 6metres.

   PQ = ST = x metres and QR = RS.

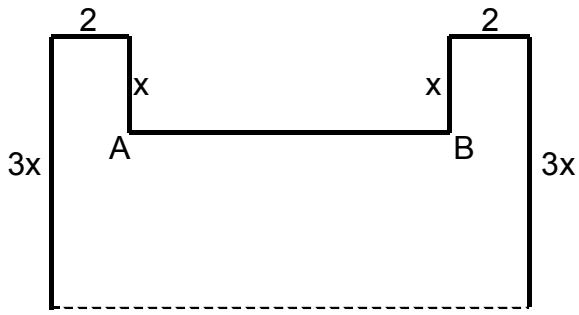
   (a) Show that $QR = 3 - x$

   (b) Hence show that the area of the flower-bed is $A = 6x - 3x^2$.

   (c) Find the value of x for which the flower-bed has maximum area and calculate the area when x takes this value.



6. The perimeter of the shape opposite is 38 metres.

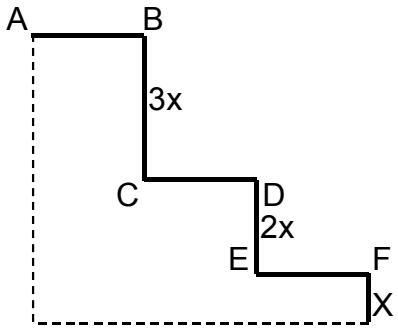


(a) Show that AB = 34 – 8x

(b) Hence show that the area of the shape is $A = 80x - 16x^2$.

(c) Find the value of x for which the area of the shape is maximised and find this maximum area.

7. A developer is designing a garden.
The wall surrounding the garden is to be of length 90 metres. The dotted lines represent a line of hedging.
AB = CD = EF.

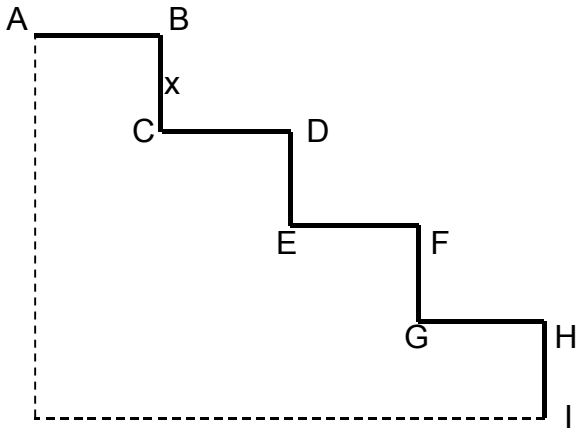


(a) Show that AB = 30 – 2x

(b) Hence show that the area of the garden is given by $A = 300x - 20x^2$

(c) Find the maximum area of the garden.

8. The diagram opposite has a perimeter of 100 centimetres.
AB = CD = EF = GH and
BC = DE = FG = HI = x.

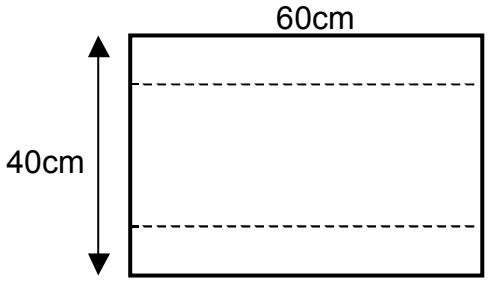


(a) Show that HG = 25 – x.

(b) Hence show the area of the shape is given by $A = 250x - 10x^2$

(c) Find the value of x so that the area is maximised and find this maximum area.

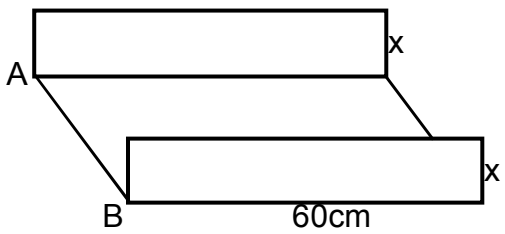
9. A rectangular sheet of metal 40 centimetres by 60 centimetres is folded as shown opposite.



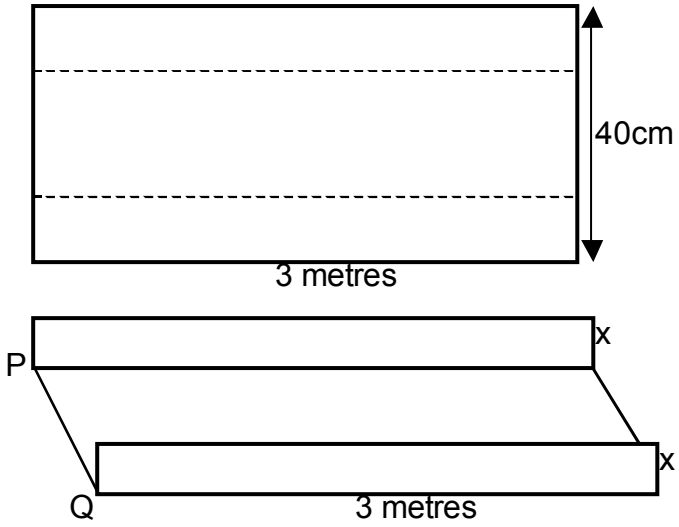
(a) Write down the length of AB

(b) Show that the volume of the shape is given by $V = 2400x - 120x^2$.

(c) Find the value of x for which the volume is maximised and find the volume when x takes this value.



10. A rectangular piece of aluminium is to be used to make some guttering as shown.

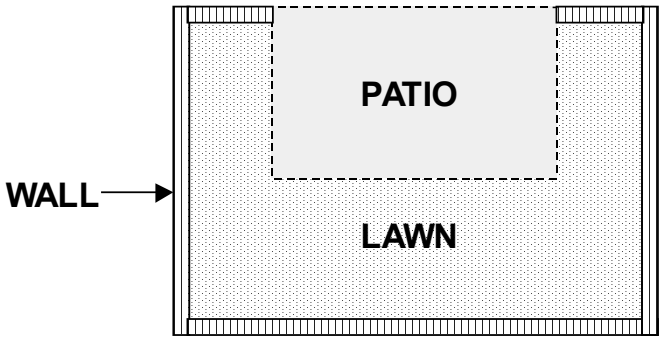


(a) Write down the length of PQ.

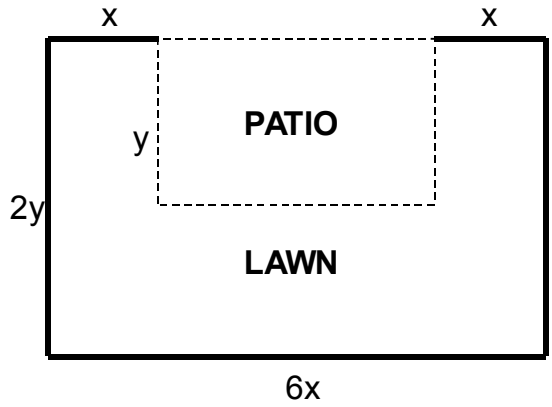
(b) Show that the volume of the guttering can be written as $V = 12000x - 600x^2$

(c) Find the maximum volume of the guttering.

11. The diagram below shows a garden with a wooden patio. A wall is built round the garden as shown



The garden has measurements as shown below.



(a) Given that the wall is 80 metres long, show that $y = 20 - 2x$.

(b) Show that the area, A, of the lawn can be written as

$$A = 160x - 16x^2$$

(c) Find the value of x which gives the lawn a maximum area.

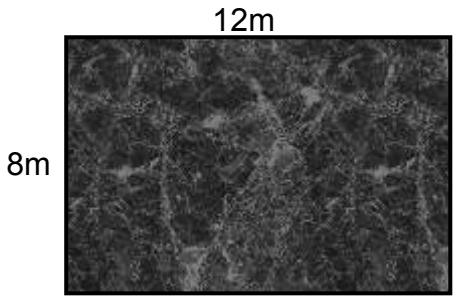
12. A rectangle has length 40 centimetres and breadth 30 centimetres.
The length and breadth of the rectangle have both to be enlarged by x centimetres.

(a) Write down the new length and breadth of the rectangle.

(b) Show that the area, A cm^2, of the new rectangle is given by
$$A = x^2 + 70x + 1200$$

(c) The area of the new rectangle must be at least ⅔ more than the original area.
Find the minimum dimensions of the new rectangle.

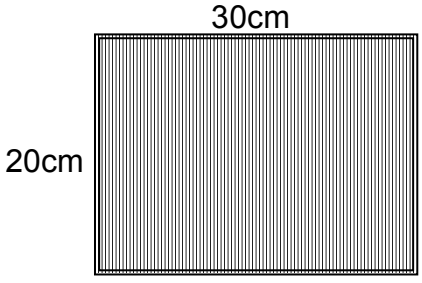
13. A rectangular shaped garden is 12 metres long and 8 metres wide.
The garden is to be made bigger. Both the length and width of the garden are to be increased by x metres.



(a) Show that the area, A, of the enlarged garden is
$$A = x^2 + 20x + 96$$

(b) The area of the enlarged garden is to be at least double the size of the original area.
Find the minimum dimensions of the new garden.

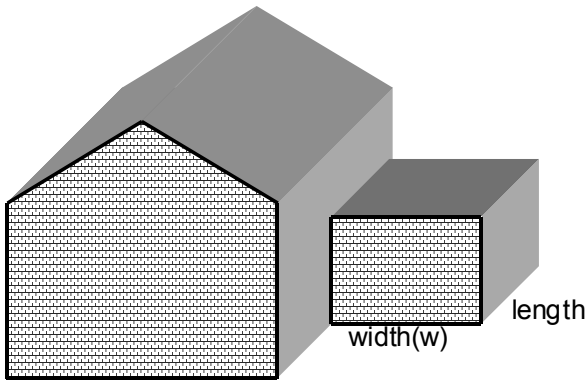
14. A rectangular wall vent is 30 cm long and 20cm wide.
It is to be enlarged by increasing both the length and width by x cm.



(a) Show that the area, A cm^2, of the new vent is
$$A = x^2 + 50x + 600.$$

(b) The area of the new vent must be at least 40% more than the original area.
Find the minimum dimensions, **to 1decimal place**, of the new vent.

15. A family want to build an extension at the rear of their house.



An architect advises that the extension should have its length 2 metres more than its width.

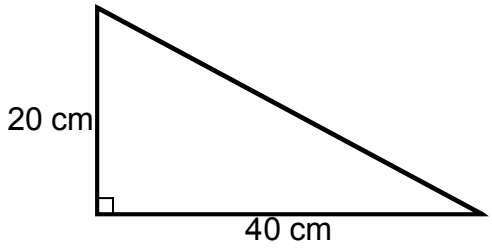
Planning regulations state that the area of the ground floor of the extension must not exceed 40% of the area of the ground floor of the original house.

(a) The ground floor of the original house is 12m by 10m.
Show that, if the largest extension is to be built,

$$w^2 + 2w - 48 = 0$$

(b) Find the dimensions of the largest extension that can be built.

16. A right-angled triangle has length 40 cm and width 20 cm.

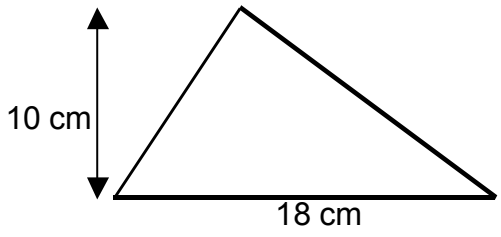


(a) The length of the triangle is increased by x cm and the width is increased by 2x cm.
Show that the area, A, of the new triangle is

$$A = x^2 + 50x + 400$$

(b) The new triangle must have an area at least 84% bigger than the area of the original triangle.
Show that the minimum dimensions of the new triangle are 46 cm by 32 cm.

17. A triangle has length 18 cm and vertical height 10 cm.



(a) The length is increased by 2x cm and the height is increased by x cm.
Show that the area, A cm^2, of the new triangle is

$$A = x^2 + 19x + 90$$

(b) The new triangle is to have an area at least 80% larger than the original triangle.
Find the minimum length of the new triangle, **correct to the nearest centimetre**.